Transient Response of a pill box cavity to a short beam pulse

Alvin Tollestrup,fnal

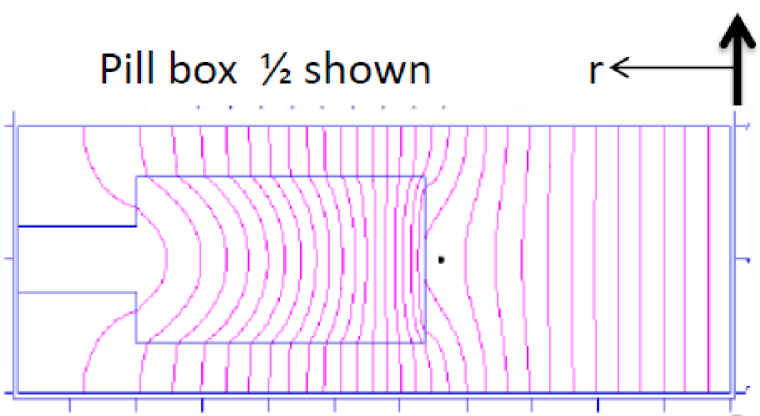
Frank Marhauser,MuonsInc

Data from simulation by Frank

- A dielectric loaded cavity with:

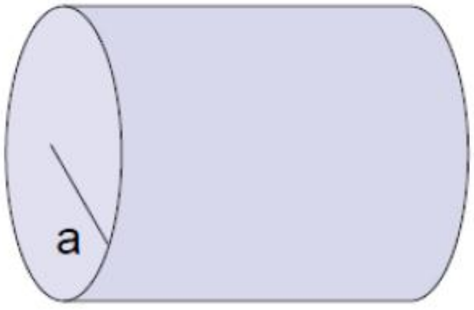
  f = 650 MHz

  Beam pulse: gaussian line charge 1 pC sigma = 1cm with v = c was sent thru the cavity on center. Cavity response was calculated for 6 cycles and Ez recorded on axis every 5 ps.



Cavity h = 2.73 cm r=12 cm ?

TM_{nmp} modes



$$E_z \sim (A\cos n\phi + B\sin n\phi) J_n(k_c\rho)\cos\left(\frac{p\pi}{d}z\right)$$

$$E_\rho \sim (A\cos n\phi + B\sin n\phi) J_n'(k_c\rho)\sin\left(\frac{p\pi}{d}z\right)$$

$$E_\phi \sim (A\cos n\phi - B\sin n\phi) J_n(k_c\rho)\sin\left(\frac{p\pi}{d}z\right)$$

Eq. for freq of mode:

$2\,\pi/\lambda = \text{Sqrt}[(b_m/a)^2 - (\pi p/d)^2]$

b_m Bessel roots

$J_n(k_{nm}a) = 0$

$p = 0,1,2,3\ldots$

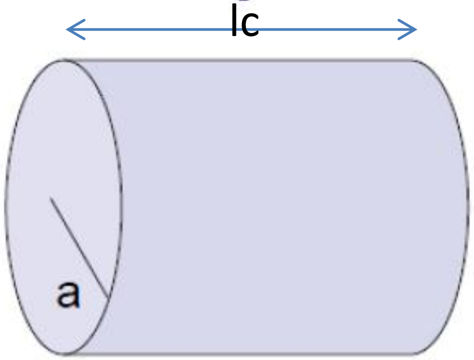
From boundary conditions.
p begins at 0.

For this talk, we use only the n=0 axisymmetric modes.

For p = 1, we must have Er= 0 on the ends so Er ~ Sin[p π z/lc] and hence Ez ~ Cos[p π z / lc] where lc is cavity length.

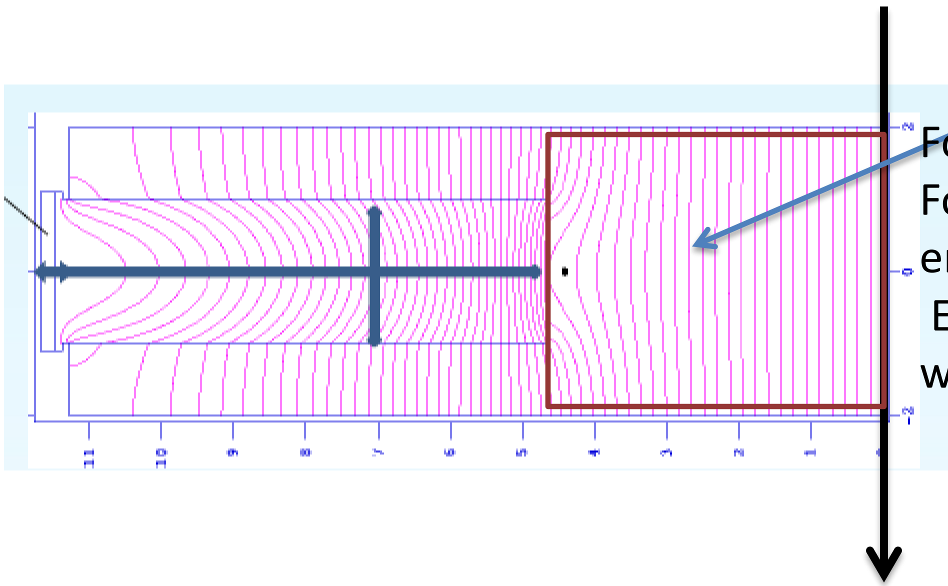
First TM cavity mode "usually" is TM_{011}.

TM_{nmp} modes



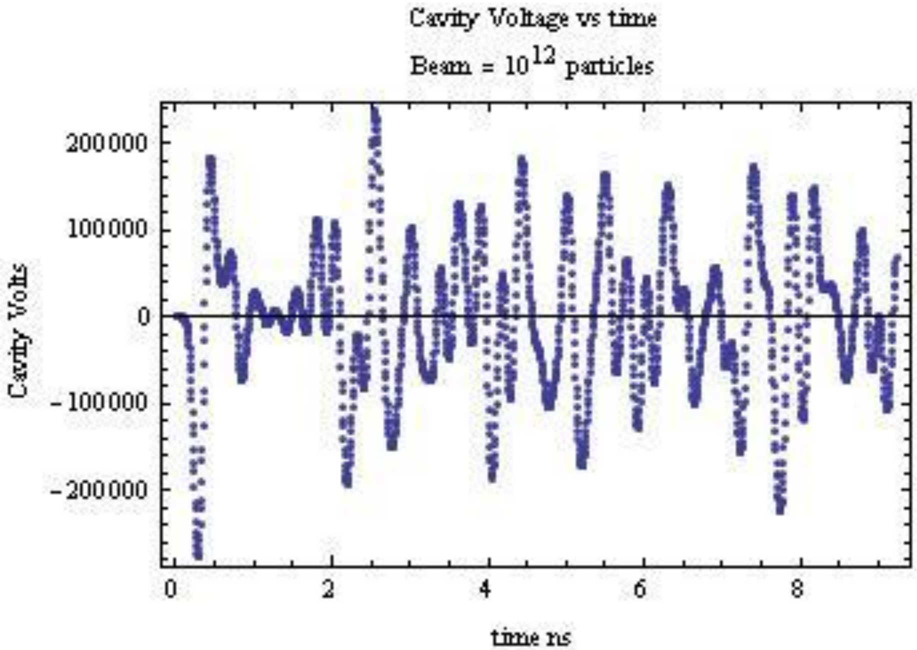
For this talk, we use only the n=0 axisymmetric modes. For p = 1, we must have Er= 0 on the ends so Er ~ Sin[p π /lc] and hence Ez ~ Cos[p π z / lc] where lc is cavity length.

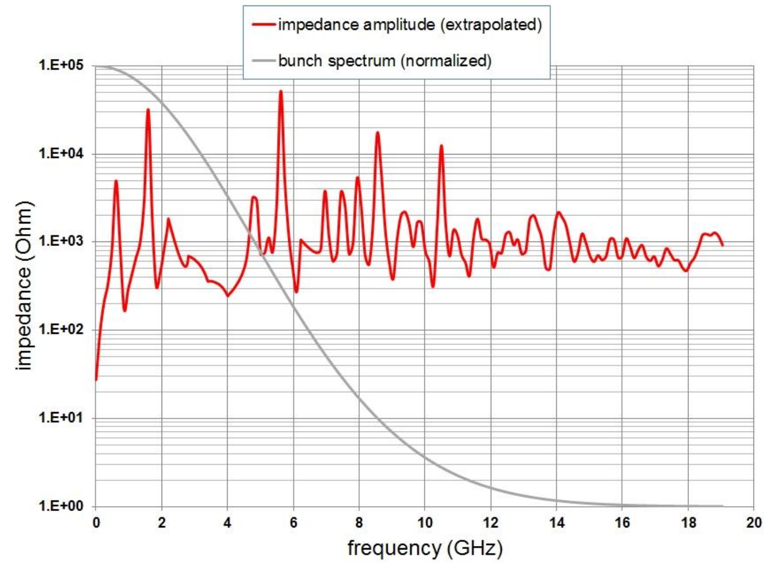
For the general case there are both Sin and Cos terms as we will see in Franks simulation.

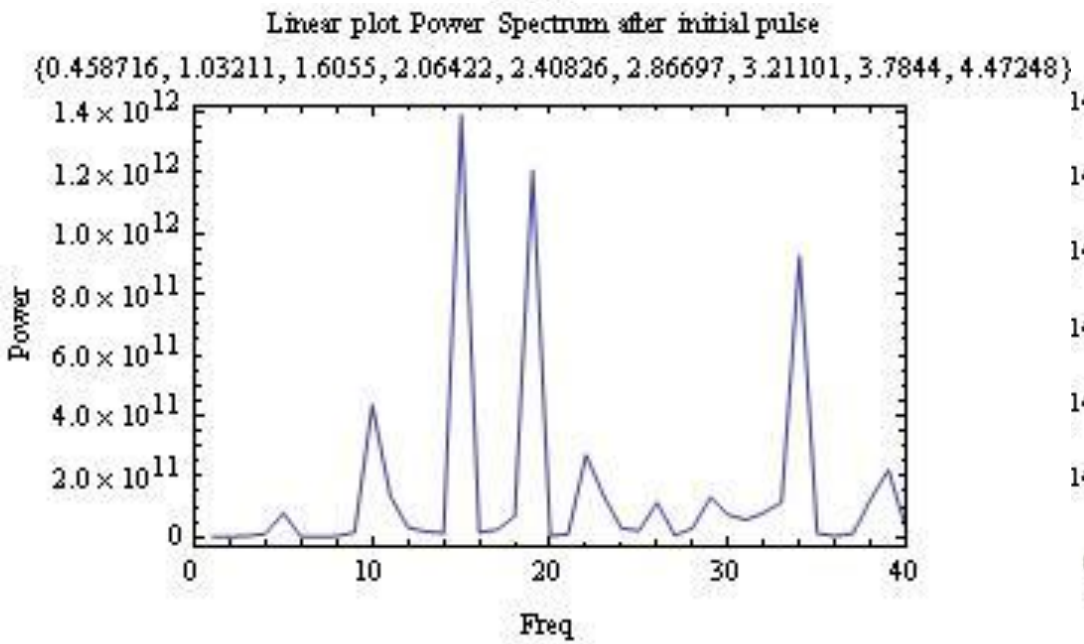


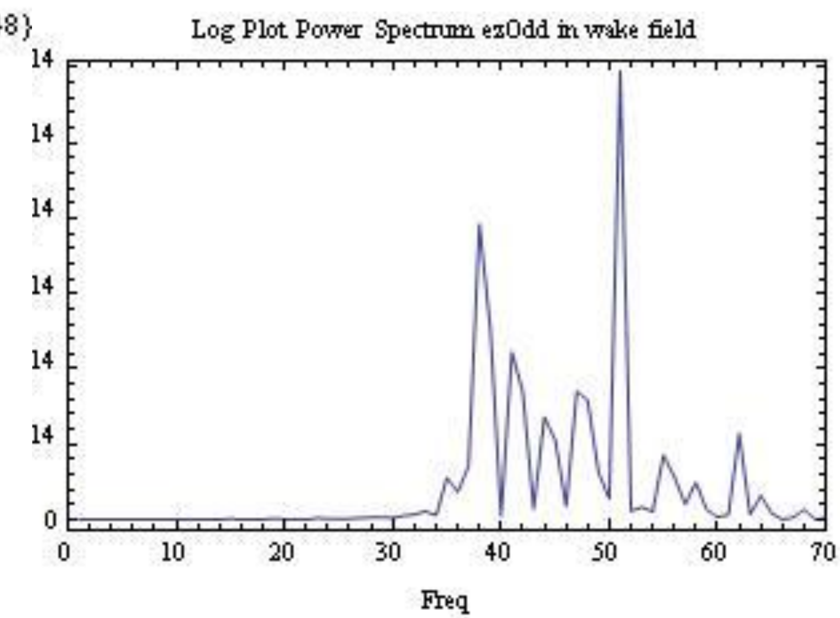
For this box
For p = 1, we no longer have Er= 0 on the end so
Ez ~A Sin[p π z/lc] + B Cos[p π z / lc]
where lc is cavity length.

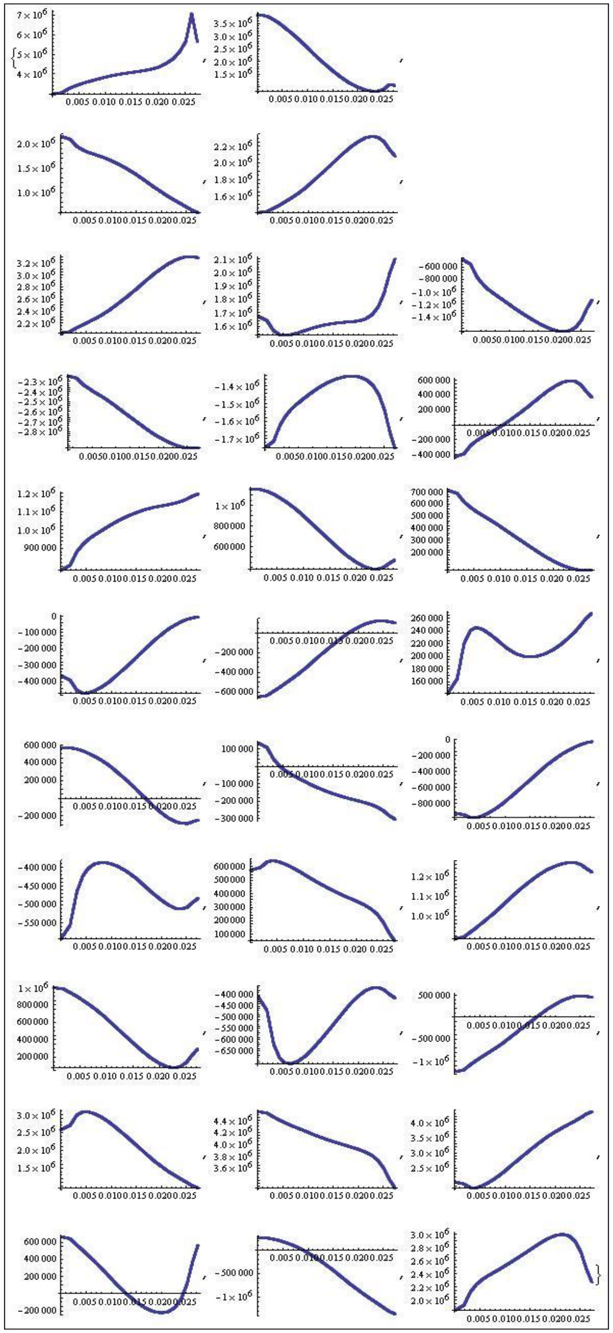
Some Results from Frank's Simulation





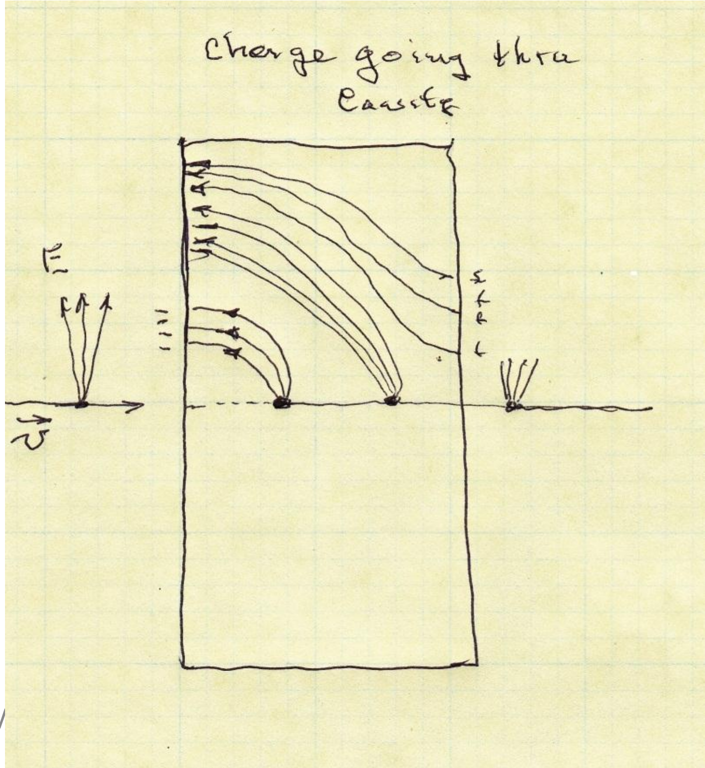


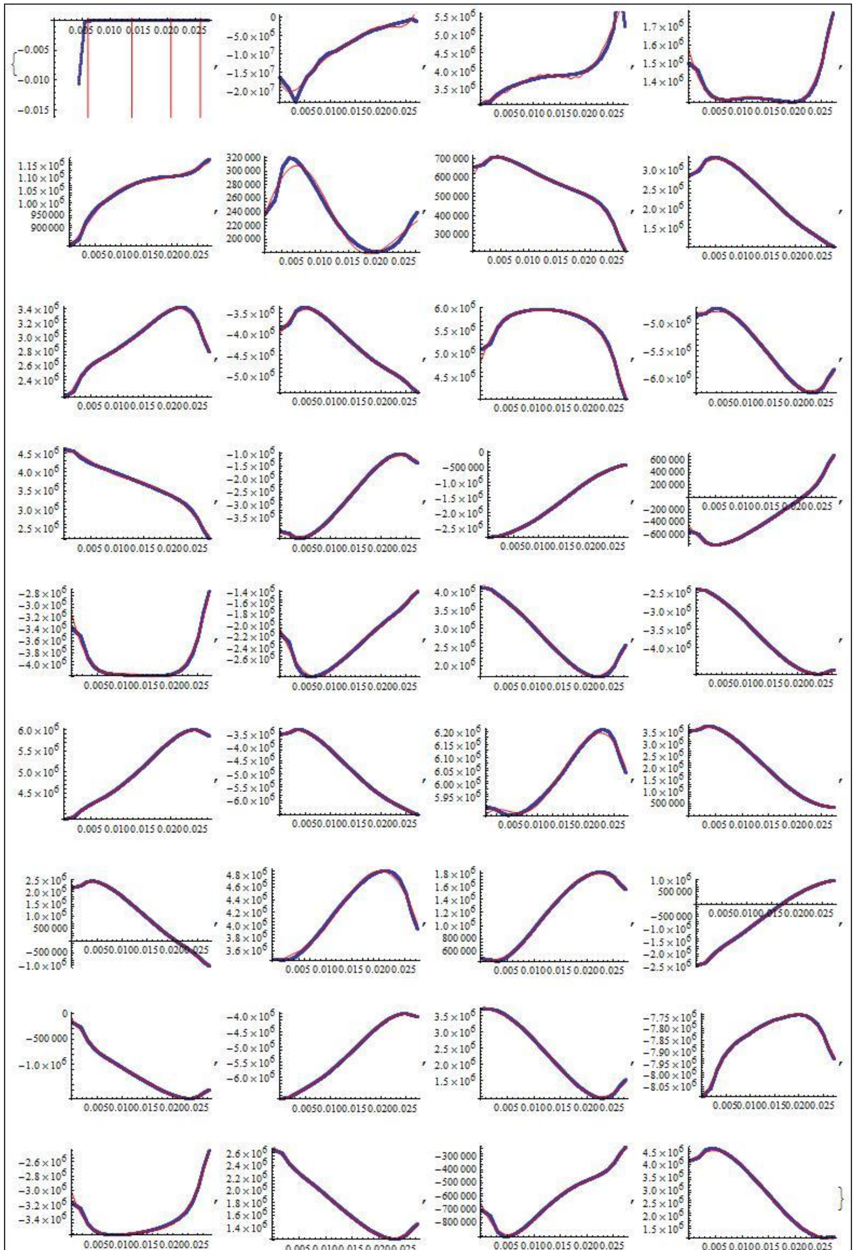




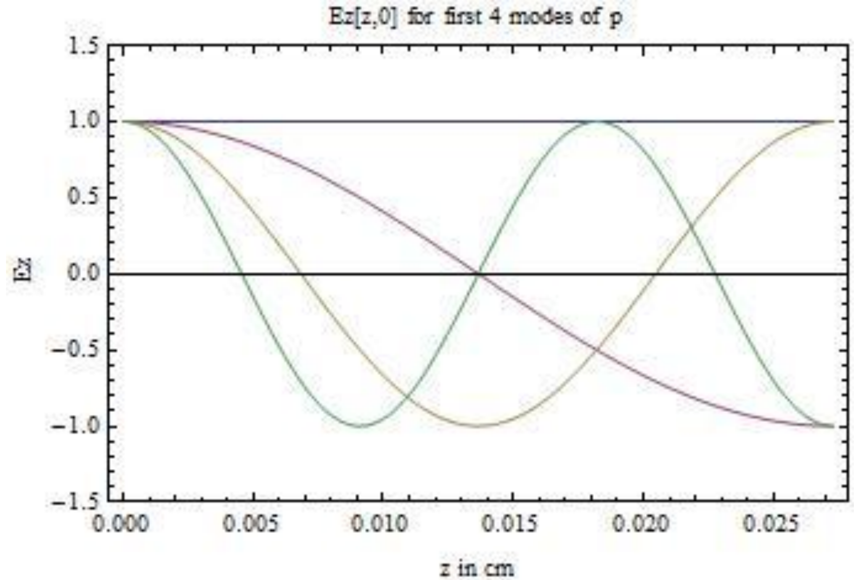
Ez[z] snapshots over 6 cycles of 650 MHz.

Ez[z,0] = Constant Cos[pπz/d] p={0,1,2,3} which leads to a high frequency modes. p=0 is the normal mode we use for acceleration. The plots at right for the fundamental mode would be constant horizontal lines at heights given by Sin[ωt] if only the p=0 mode was excited. The z variation shows directly that the higher p modes were excited.





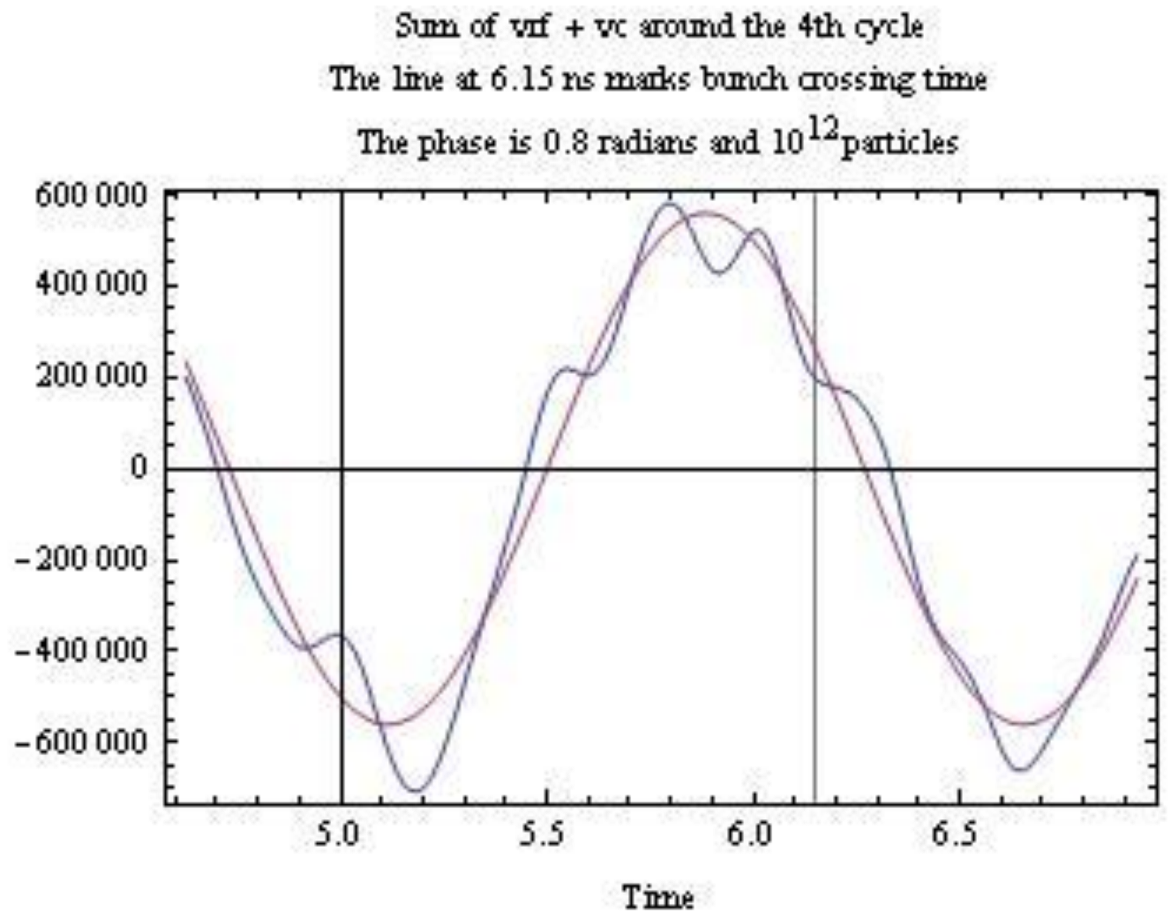
Ez vs z fits every 500 ns using
4 terms of Cos[p Pi z/d],{p= 0,1,2,3}



The light solid line shows the results of fitting with the above set of functions.

Note: the modes 1, 3 are odd and
modes 0, 2 re even.

The plot below shows the total voltage across the cavity 4 cycles after a bunch of 10e12 particles pass thru. The voltage of the wake field was determined by integrating Ez[z,0] across the cavity. This is the voltage that the second bunch in the train would see. After 10 bunches pass thru the cavity the 11 bunch would see the sum of the previous 10 wake fields. They are not harmonically related and so one might guess that the answer would be about Sqrt[10] worse.



Analytical Simulation of Gaussian Beam Pulse going thru A Pill Box

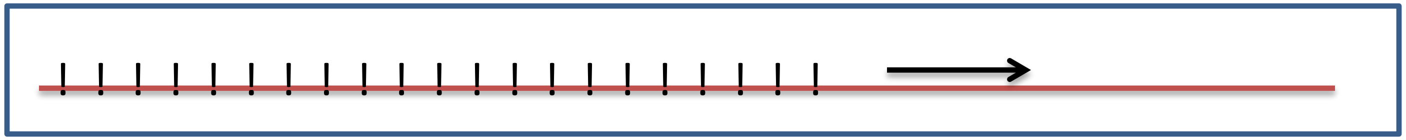
We simulate of the wake field for a time corresponding to 21 bunches after a beam bunch goes thru the cavity. Spacing between bunches = 4 cycles.

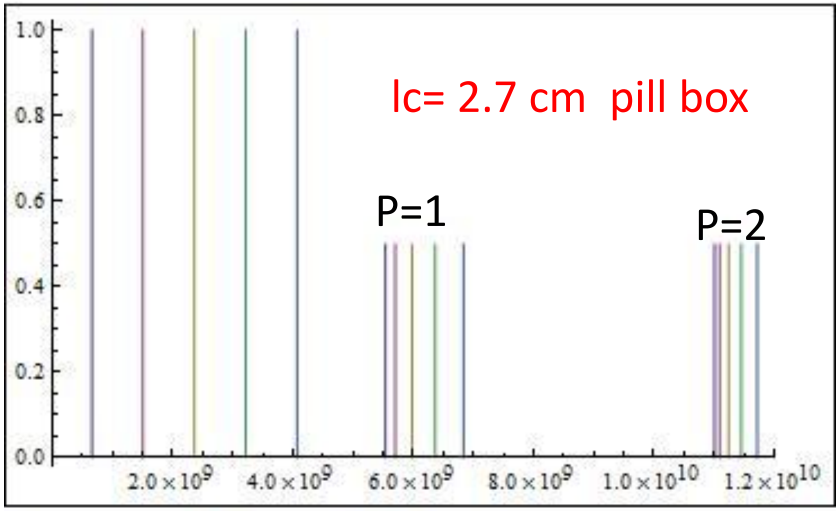
We use a 650 MHz pill box with a beam:

Nb = 21 bunches
Sigma z = 2 cm
Muon/bunch 10 e 11
Cavity length lc between 2 and 10 cm
n = 1, 2, 3 are the radial modes
p = 0, 1, 2 go with Sin/Cos of (p π z/lc) are the z modes.
v = c in the mathematical solution

Wakefield definition: The energy a particle following the generating bunch by a distance z receives from the residual field in the cavity. It includes the transit time effect and is not the voltage across the cavity. See Gregory R. Werner arXiv:0906.1007v 1 {physics.acc-ph] 4 June 2009.

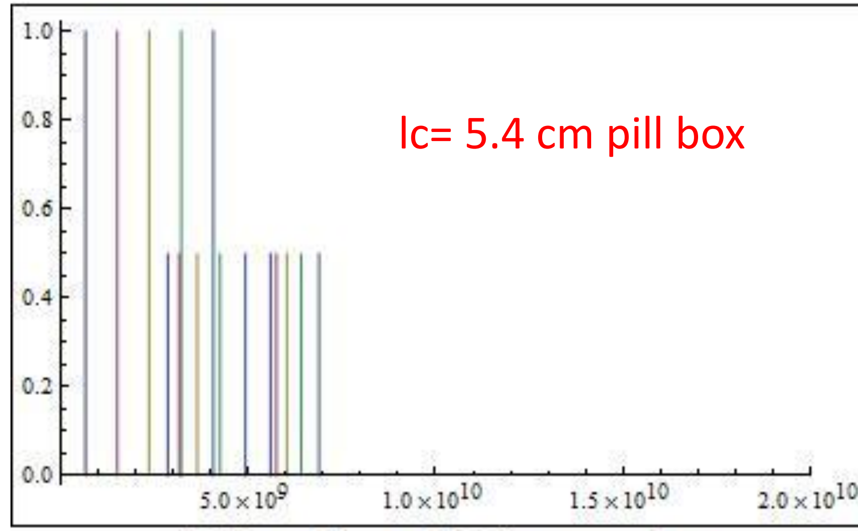
The last bunch sees the sum of the wake fields of the previous 20 bunches



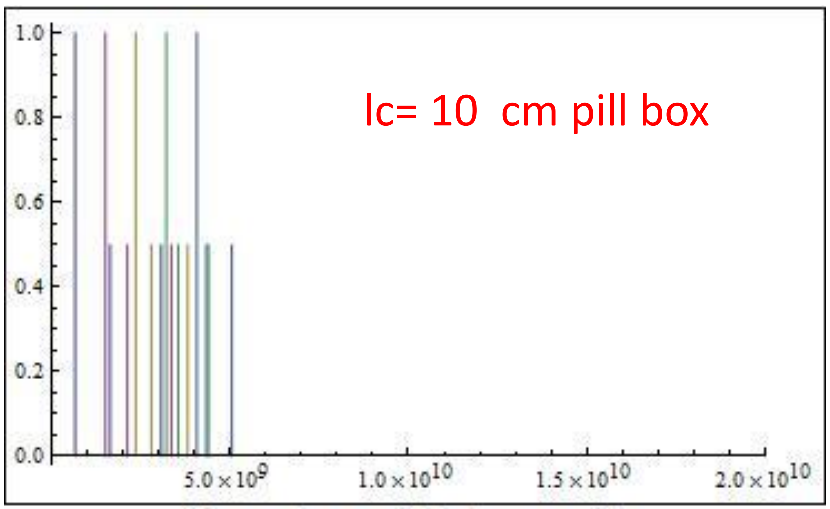


2.73 cm long pill box cavity

Showing n=0 with {k=1,2,3,4,5} and p= 0,1,2



5.4 cm long plii box cavity



10 cm long pill box cavity

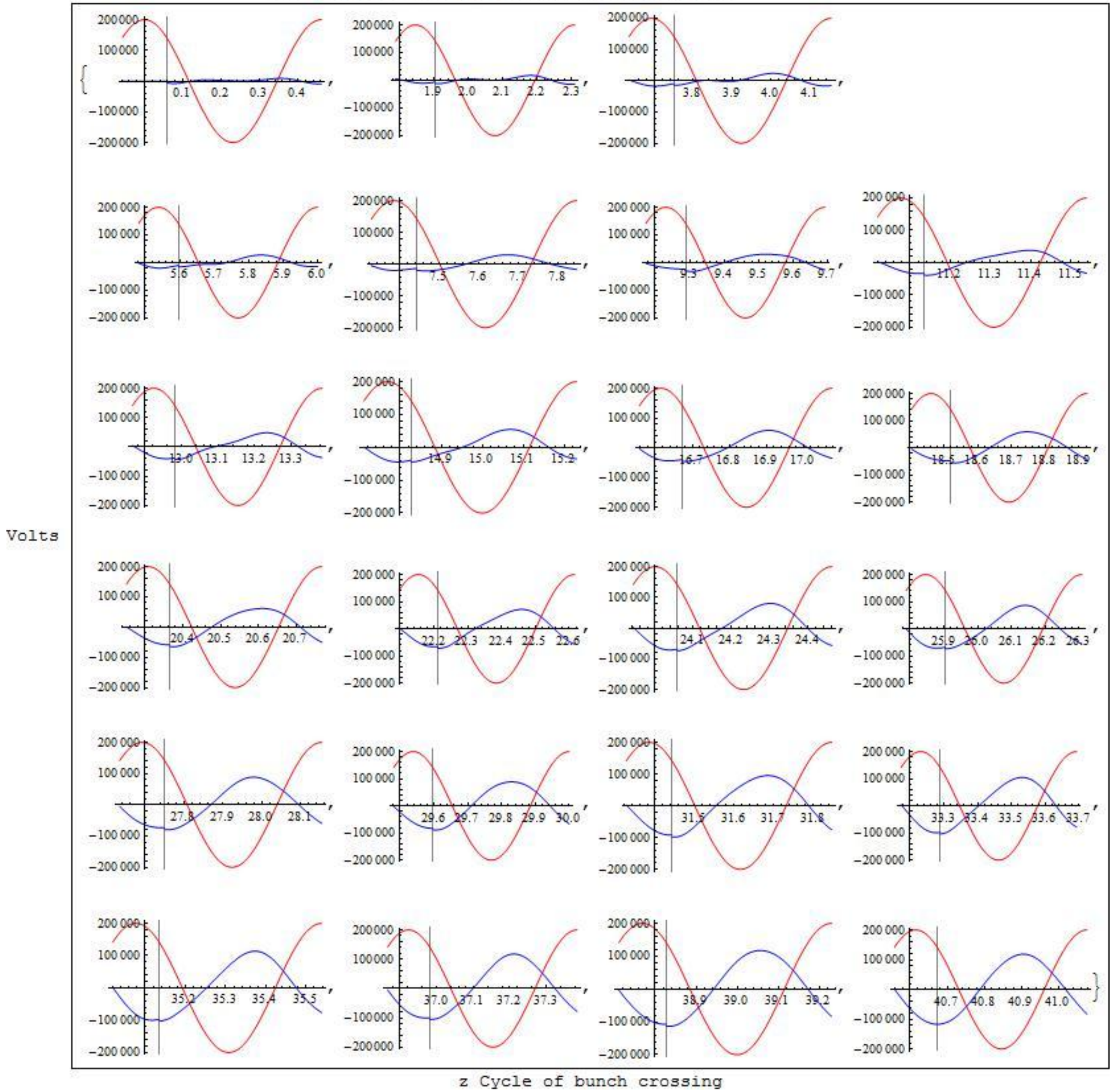
Example spectrum of pill box cavities of different length. For longer cavities, the z dependent modes move down to lower frequencies. Note lc = length of pill box. This doesn't change the frequence of the p=0 modes.

Bunch ΔE from wakefield alone after n = {1,nb} previous bunches

n=3, p = 0, nb=21

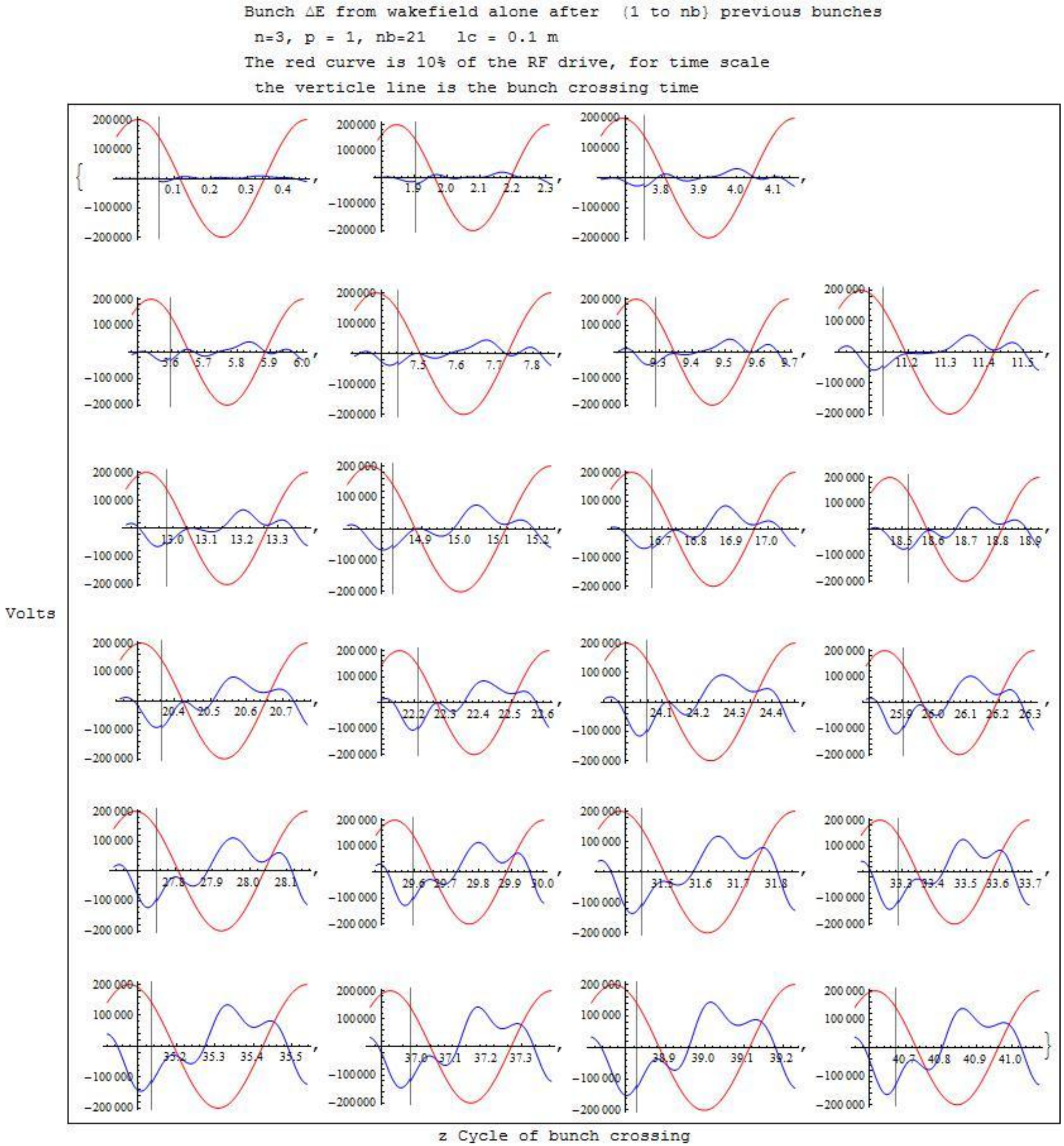
The red curve is 10% of the RF drive, for time scale

the verticle line is the bunch crossing time



Lc = 10 cm, n= 1,2,3 p=0

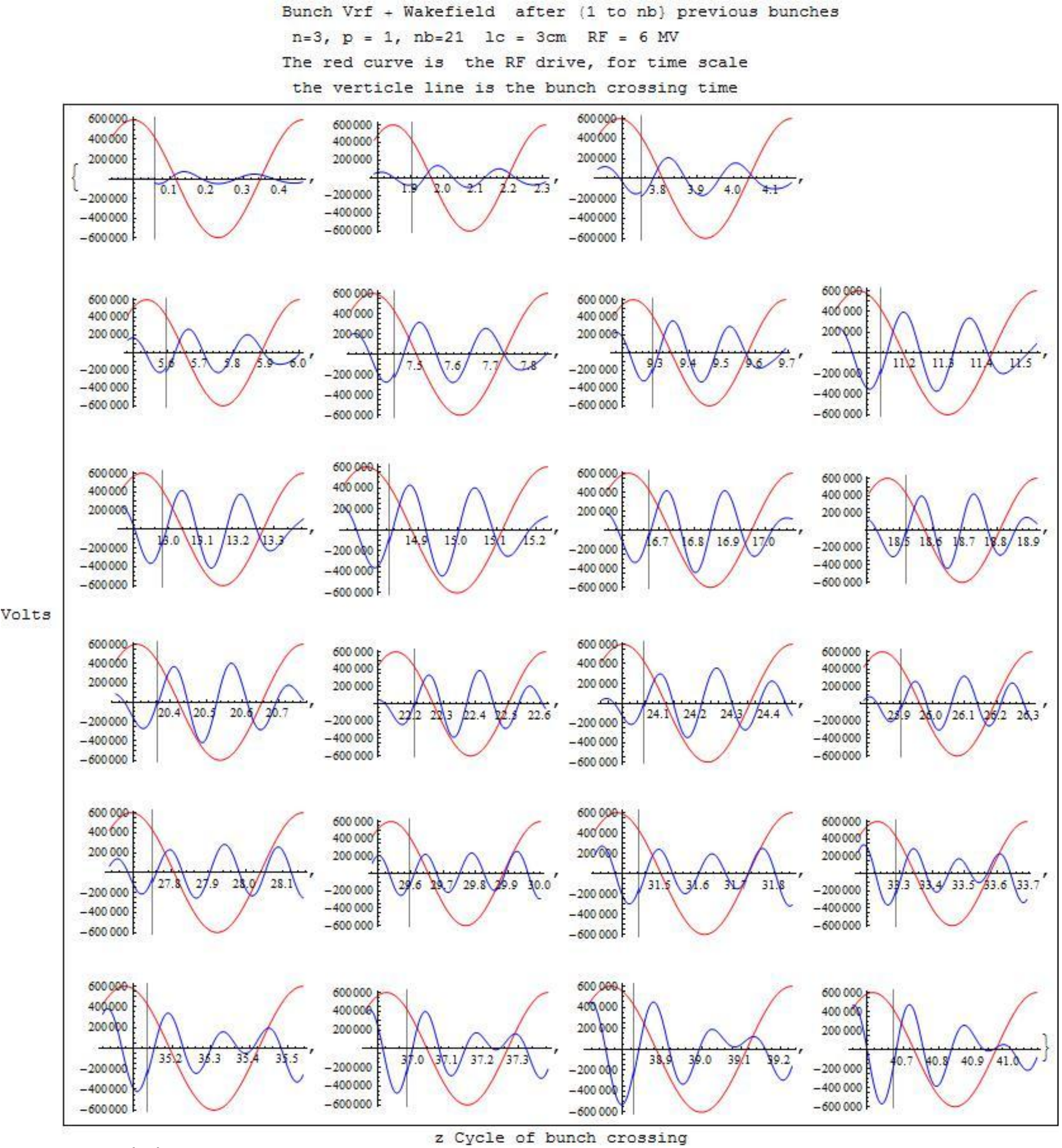
The red curve is 10% of the RF drive voltage for a gradient of 20 MV/m. The blue curve is the induced wakefield that each succeeding bunch sees as it crosses the cavity. The fundemental mode term adds coherently and dominates at the end. The n=1 and 3 modes add but are incomensurate in frequency and so give a varying resultant voltage.



Lc = 10 cm n = 0,1,2 p=1

P=1 so the z modes are present. Compare this to previous slide to see the effect of the p=1 modes.

The vertical line marks the point where the center of the bunch will cross the cavity. I have picked a phase of – 45 degrees.



Lc = 3 cm n = 0,1,2 p=1

In this plot, the gradient is still 20 MV/m but the cavity voltage is 0.6 MV and is plotted full scale! Note that for this short cavity that the p=1 modes are making the dominant effect and some way must be found to damp them.
Also note the beating between these modes. There are places where the net wake field is almost zero.

How does mode freq vary with cavity length?

n	p	Freq	n	p	Freq
1	0	6.4955×10^{8}	1	1	1.50037×10^{10}
2	0	1.49099×10^{9}	2	1	1.50636×10^{10}
3	0	2.3374×10^{9}	3	1	1.51707×10^{10}

Table of frequencies for lc = 0.01cm

n	p	Freq	n	p	Freq
1	0	6.4955×10^{8}	1	1	7.52289×10^{9}
2	0	1.49099×10^{9}	2	1	7.64167×10^{9}
3	0	2.3374×10^{9}	3	1	7.85082×10^{9}

Table of frequencies for lc = 0.02cm

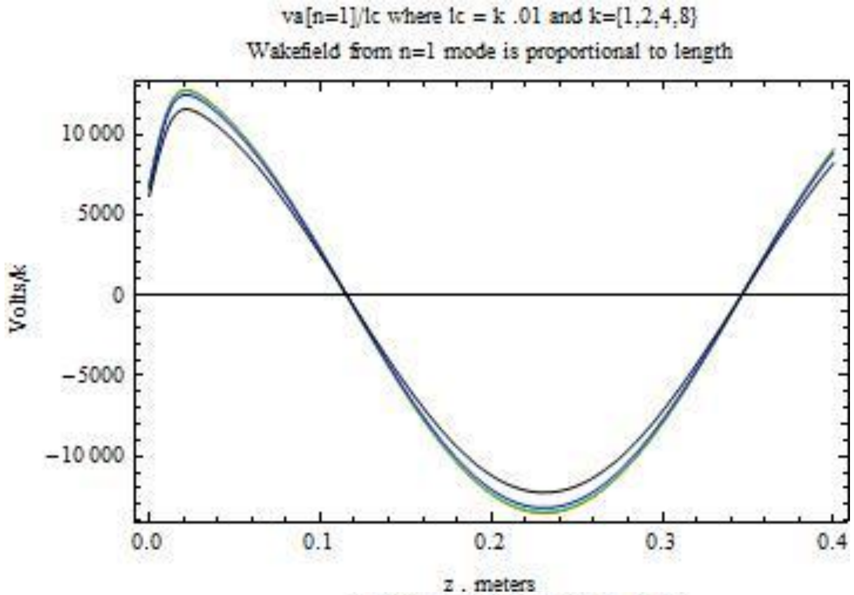
n	p	Freq	n	p	Freq
1	0	6.4955×10^{8}	1	1	3.80328×10^{9}
2	0	1.49099×10^{9}	2	1	4.03312×10^{9}
3	0	2.3374×10^{9}	3	1	4.41661×10^{9}

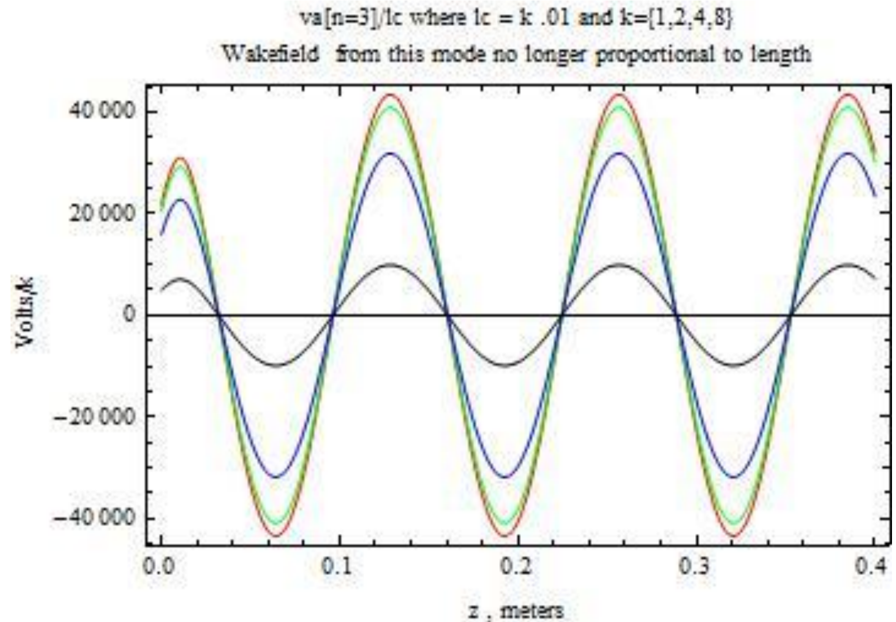
Table of frequencies for lc = 0.04cm

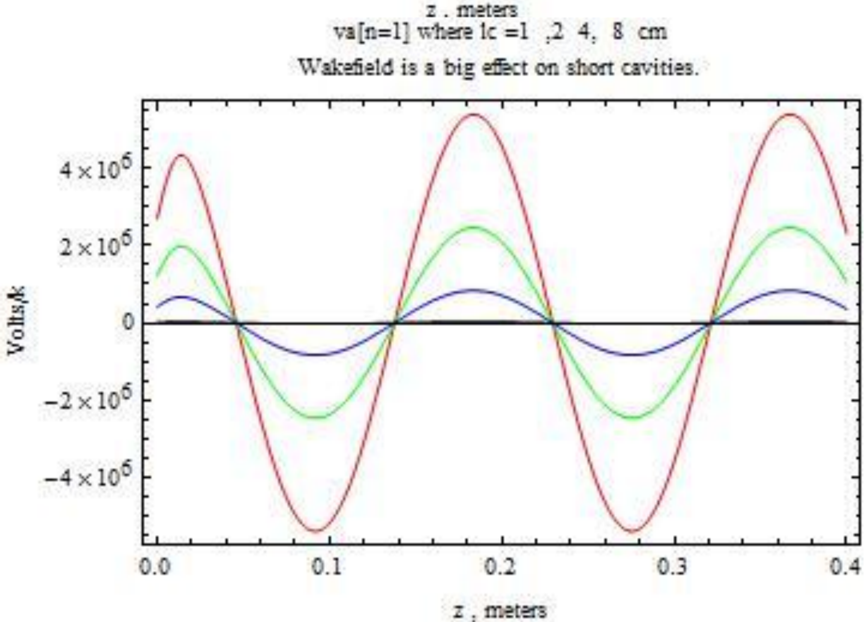
n	p	Freq	n	p	Freq
1	0	6.4955×10^{8}	1	1	1.9831×10^{9}
2	0	1.49099×10^{9}	2	1	2.39453×10^{9}
3	0	2.3374×10^{9}	3	1	2.99569×10^{9}

Table of frequencies for lc = 0.08cm

How does mode voltage vary with cavity length







Consider a cavity as an R L C circuit. The beam passing thur depisits charge q on the capacitor C starting a transient with amplitude Vrf = q/C.
For a pill box, the equivalent inductance is just proportional to length. Since the frequency is invariant, the equivalent C must vary as 1 / lc. So for a fixed beam charge the mode voltage would be proportional to 1/ lc.

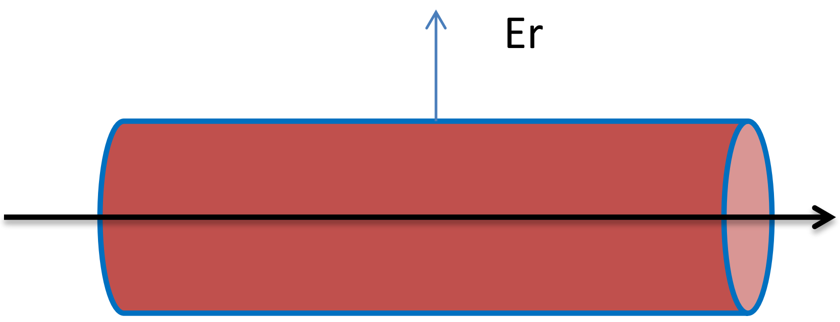
Bunch-gas interaction: some thoughts

Alvin Tollestrup

11-7-2013

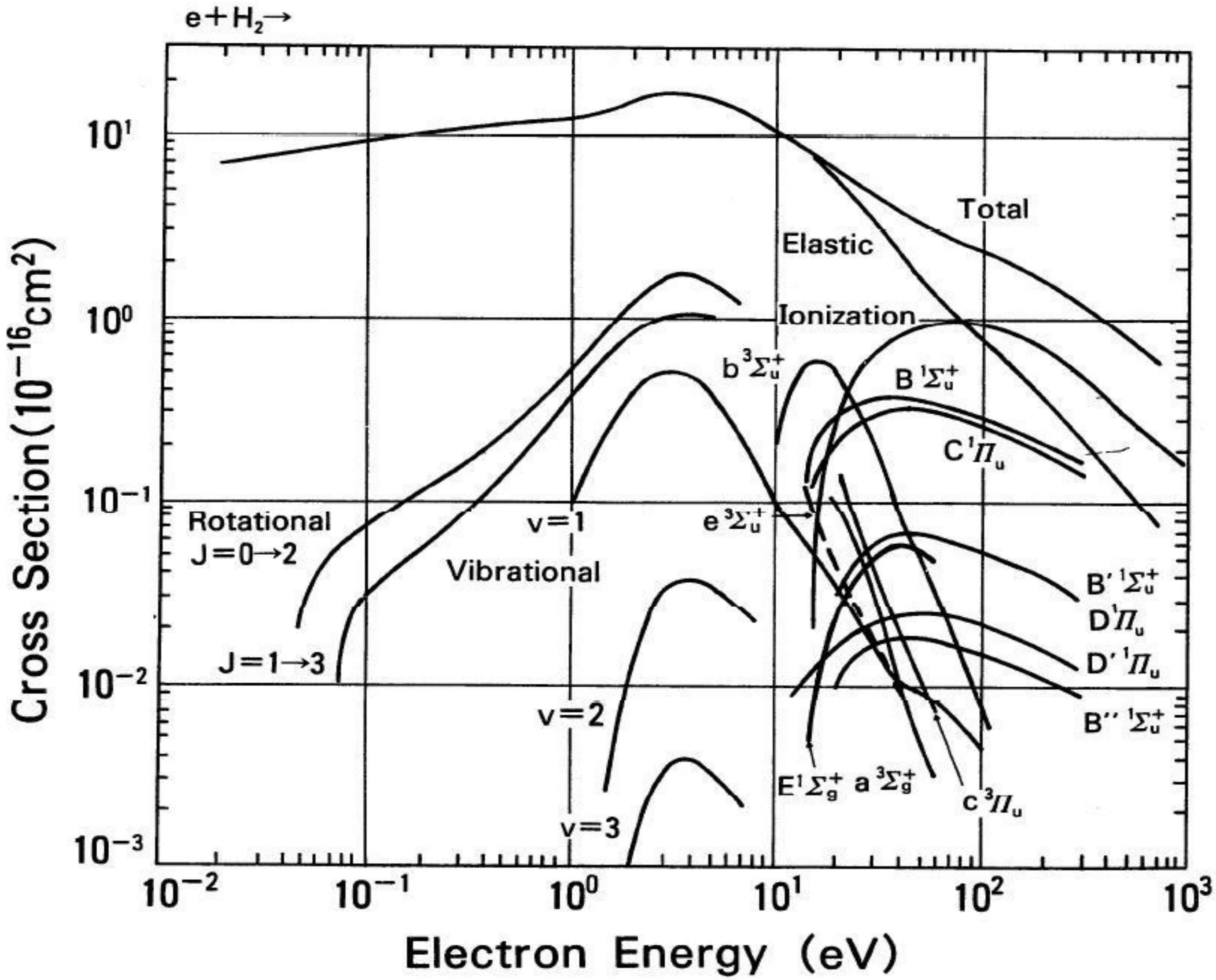
Model Beam Bunch-gas interaction

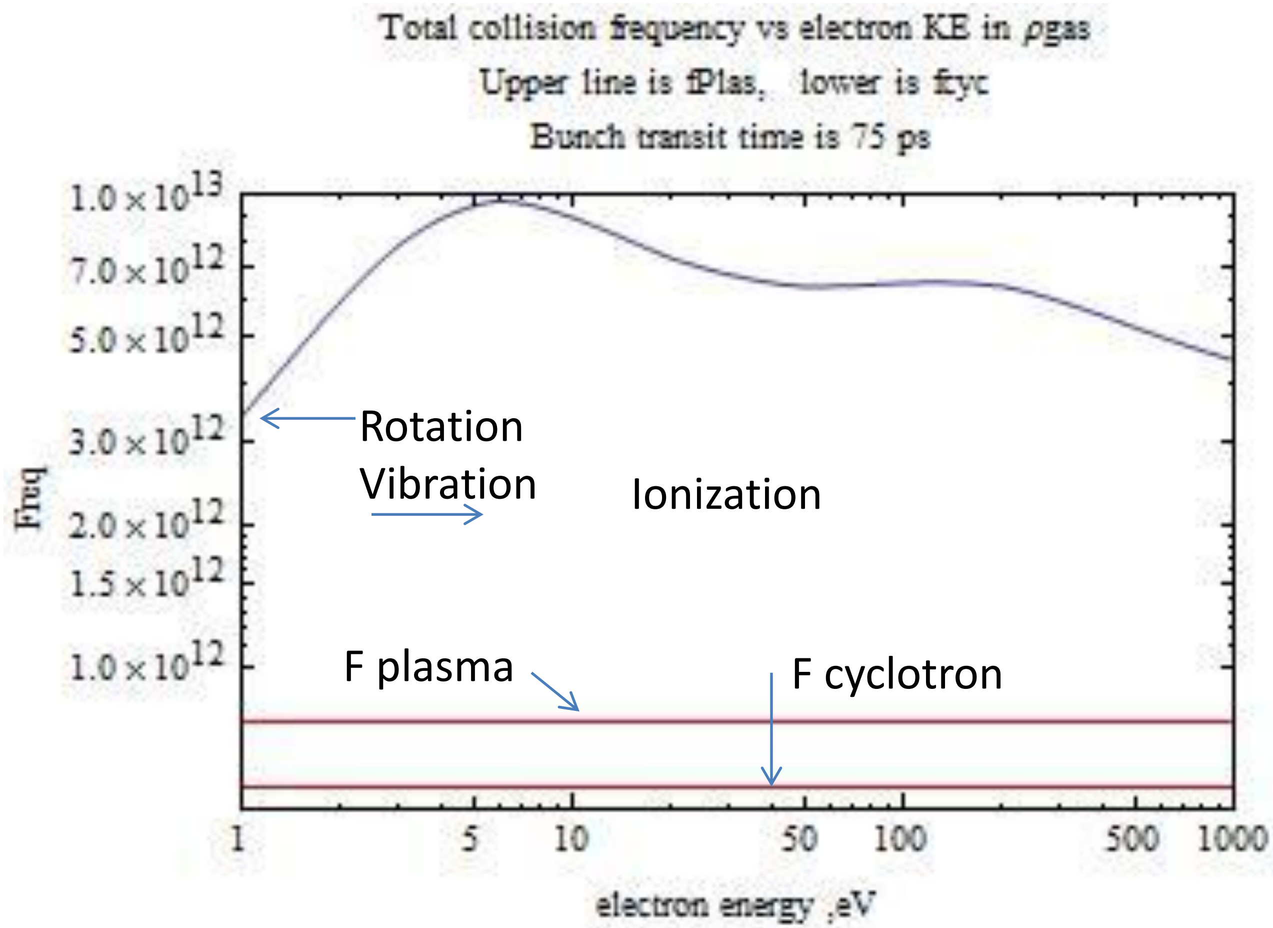
- 1. Bunch 2 cm long 1 mm radius 200 MeV . Transit time 75 ps
- 2. Bz = 20 T. pGas = 2600 psi
- 3. density H2 mol = 4.75 10 e21
- 4. density ions = 3.12 10 e15
- 5. Cyclotron F = 5.6 10 e11
- 6. Plasma F = 7.57 10e11
- 7. Er , z=0, r=1mm = 5 MV/m
- 8. Bϕ, z=0,r=1mm = .15 T



Moving with beta = .88

Electron – H2 cross sections

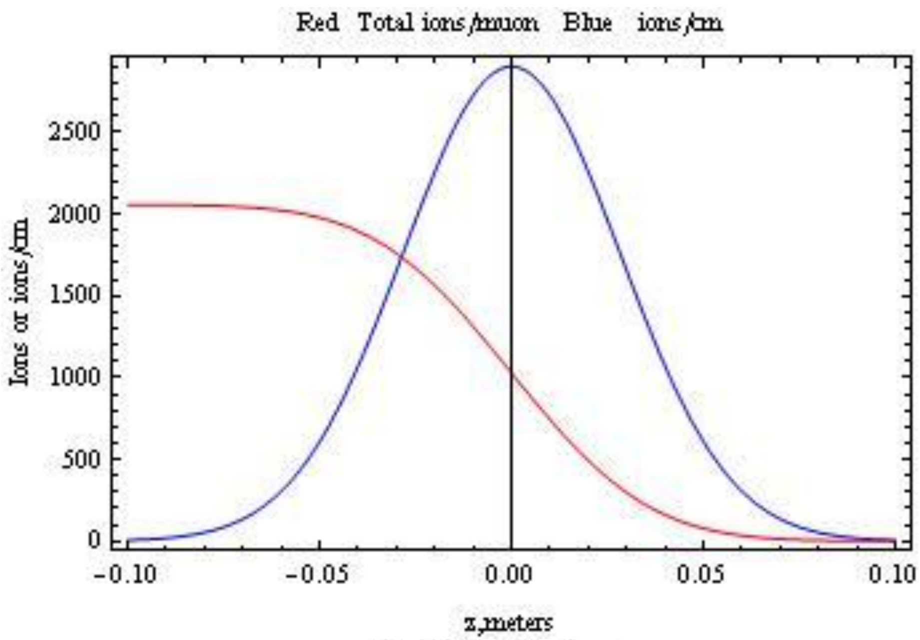




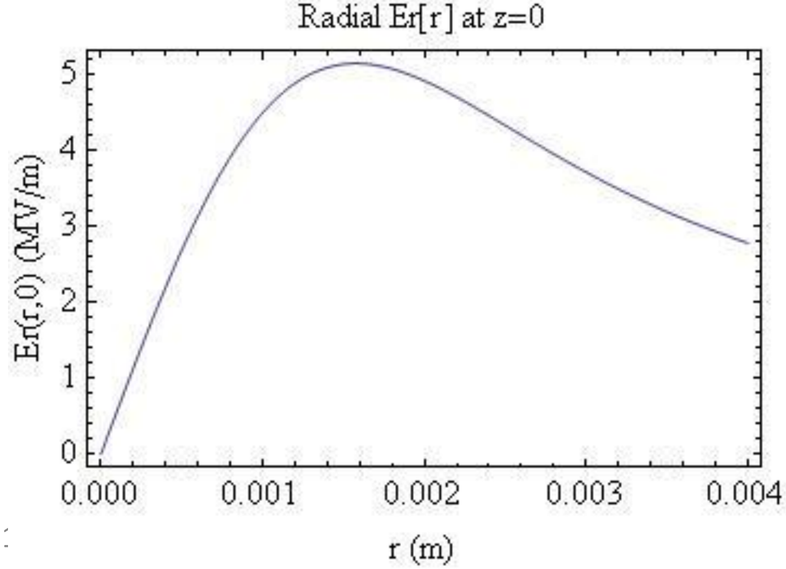
Some times: 10^11 muons/bunch

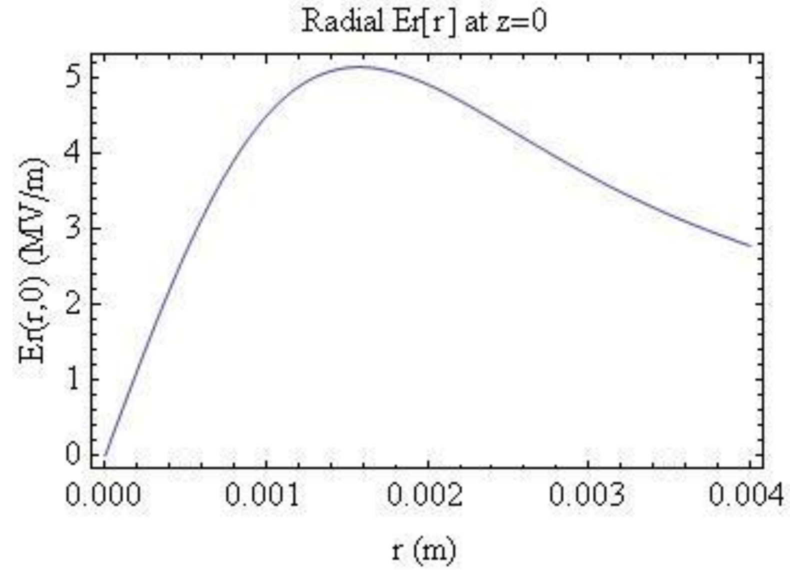
1. Collision time 0.1 ps
2. Bunch transit time = 2 sigmaz / v = 75 ps
3. Drift velocity of electron in Pgas = 2600,E= 5 MV/m E/P = .37, drift velocity of an electron at this E/P = .5 10 e6 cm/sec or 0.5 microns/ps
4. Life time of electron with 0.2% 0xygen ~ 10 to 100 ps

Question: Does the plasma neutralize the space charge?



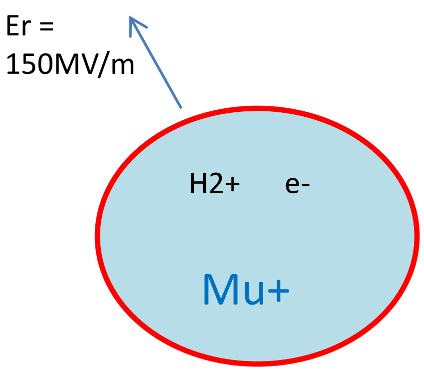
The red curves shows the total number of ions at any given point along the length of the bunch with no capture to form O_2^{--} .
Below ar plots of the self field of the muon bunch



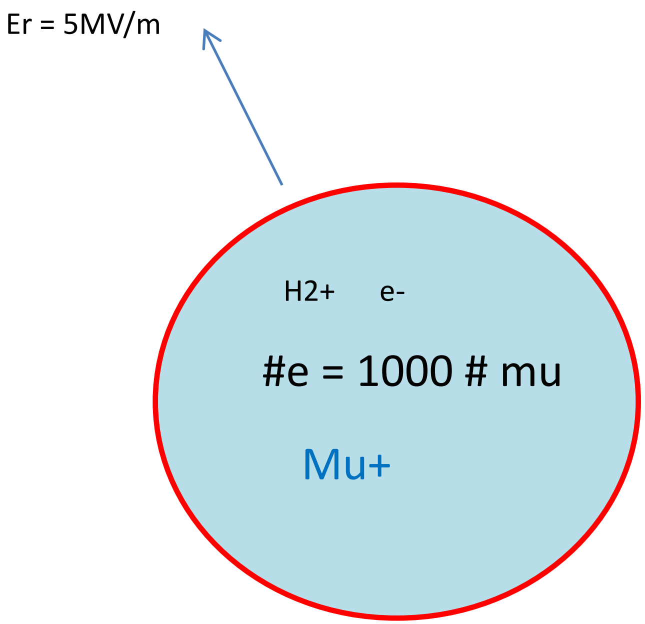


How about space charge neutralization?

1. Like electron cloud in accelerators. The beam makes a plasma e- and H2+ around the beam. Consider mu+ beam. The muons pull in the electrons, neutralize the electric field and the remaining B field focuses the beam. Or else there is an interaction between the cloud plasma and the bunch that causes blow up of the emittance. Is such a thing possible with an intense beam pulse in H2?
2. Two facts:
    1. Each muon makes 1000 ion pairs/cm of path. There are hence 1000 times as many + ions and 1000 times as many electrons / cm as there are beam charges.
    2. The resulting plasma frequency is very high … of the order of 10 e12 Hz



In the 2 mm circle there are 1000 e and H2+ for each muon. The Plasma will try and neutral the field of the muons



But can the electrons move fast enough? Their velocity is of the order of 0.5 microns/ps. Since the bunch passage is inly 75 ps we would like to to see neutralization take place at a given point in the order of 1 ps. The outside electrons have to move 2 mm/1000 or 2 microns. Thus they neutralize in about 4 ps. Bunches of 10e12 neutralize much faster and the fields are much larger.

What is equilibrium state?

Inside the electrons move slightly inward . The positive ions are very heavy and essentially stay put. Since the mu Er field is proportional to the radius, the motion of the electrons is proportional to r also. The whole cylinder of electrons contracts. If it shrinks by 1/1000 its density will increase by 1/1000 and will have:

Density mu+ + density H2+ =density e-

This leaves a ring around the outside of H2+ . The field from this ring is =0 on the inside and 5 MV/m on the outside.

This leaves the azimuthal B field that is a positive focusing force.

Some Questions to answer with good simulation

1. Is there a coherent energy loss from the bunch that adds on to the dEdx loss by ionization?
2. Are there plasma modes that can interact with the bunch phase space distribution?
3. Our measurement of the electron capture time indicates the time will be less than 1 ns, even as short as 0.1 ns. This time depends on the plasma temperature. We think this time is very short because of the high collision frequency. Are we missing something?

4. The arguments here would say the effects are small. However the simulation must be sufficiently detailed, including high collision frequency to answer some of these detailed questions. The first two questions are of beam dynamics, the last concerns ion chemistry.

Conclusion

1. Wake fields are important for the case where we are using 650 MHz or higher when the bunch size gets of the order of 10e11.
2. Simulation must include p=1   z modes. Watch out for short bunches, sigmaz < 2 cm and short cavities. In linacs higher modes are syphoned off and damped. This is going to be very hard in a helical channel
3. It looks like the bunches are very well protected from the type of plasma oscillations that take place in linacs and synchrotrons.
4. See next talk by Moses on first attempt using WARP. Simulating the plasma embedded in a high density neutral gas is going to take some experts. Roman is making progress on this problem, We will have to see if his simulation can handle both the chemistry problem in the plasma that determines the electron lifetime as well as the dynamic effect on the muons from their self field.